EMITTER International Journal of Engineering Technology
Vol. 14, No. 1, June 2026, pp. 121~141
DOI : 10.24003/emitter.v14i1.1017

An iterative LQR control method for 6-DOF UAV quadcopter under disturbance and payload variation

Vo Van An[1], Van Binh Nguyen[2]

[1]Institute of Engineering Technology, Thu Dau Mot University, Ho Chi Minh, Vietnam
[2]School of Electrical Engineering, International University, Vietnam National University Ho Chi Minh City, Vietnam
Corresponding Author: nvbinh@hcmiu.edu.vn

Received January 4, 2026; Revised March 5, 2026; Accepted June 8, 2026

Abstract

This paper proposes an optimal control method based on the iterative LQR aimed at enhancing the trajectory tracking accuracy of a 6 DOF quadcopter UAV under external disturbances and varying payloads. The main distinguishing feature of the proposed method is the iterative optimization mechanism of the weighting matrices Q and R through the logarithmic normalization of error metrics, including ITAE, ISE, and RMSE, which automatically adjusts the controller based on the system's actual performance. The effectiveness of the method is evaluated through simulations in MATLAB across various flight trajectories and scenarios, including ideal conditions, disturbances, and combined disturbances with varying payloads. The simulation results show that the iterative LQR method significantly improves control quality compared to traditional PID and LQR controllers, with error reductions ranging from 6.6% to 41.8%, particularly evident in the x and y position axes and the yaw angle ψ. Furthermore, preliminary experimental results on quadcopter UAV hardware show that the proposed iterative LQR algorithm helps improve control quality and enhance flight attitude stability under low wind disturbance and varying payloads. These results demonstrate the adaptability and robustness of the proposed method under the complex operating conditions of UAVs.

Keywords: UAV quadcopter, Linear Quadratic Regulator (LQR), PID control, Optimal Control, Iterative optimization.

1. INTRODUCTION

UAV quadcopters have become essential in many applications, such as surveying, environmental monitoring, and air traffic. However, in real-world environments, quadcopters often face external disturbances such as wind, changes in gravity, and other unforeseen factors. Therefore, developing effective controllers is essential to ensure the stability and performance of UAVs. One of the biggest challenges in controlling quadcopters is reacting quickly to these disturbances. In recent studies, various control methods have been proposed, such as PID controllers [1-8], fuzzy logic control [9-10], backstepping control [11], fuzzy-PID control [12-14], model predictive control [15], linear quadratic regulator [16], sliding mode control [17,18], optimal control [19], nonlinear robust control [20], neural network-based control [21, 22], PID control, and integral state feedback [23]. Among them, the optimal linear controller LQR is highly regarded for its ability to ensure stability, fast response, and low computational cost. However, most current studies use fixed weighting matrices Q and R or those selected based on experience, leading to limitations in adaptability when operating conditions change.

This study presents the design and performance evaluation of a six degrees of freedom (6-DOF) control system for UAV quadcopter, using iterative LQR control method based on iterative optimization of Q and R weight matrices, aiming to ensure accurate trajectory tracking control and stability under various operating conditions, including external disturbance and changing payload. Unlike traditional methods that often assume an ideal environment or use controllers with fixed coefficients, this research conducts a comprehensive comparative analysis between the iterative LQR and classical PID/LQR controllers through multiple complex scenarios in simulation. To enhance practical applicability, an experimental UAV platform was also developed to evaluate the performance of the control system under real operating conditions.

The structure of this article includes six main sections. Part 1 - Introduction. Part 2 - Related Works. Part 3 - Originality. Part 4 presents the complete nonlinear/linear 6-DOF UAV quadcopter dynamics model, considering the influence of external disturbances, and proposes an iterative LQR controller design for the 6-DOF UAV quadcopter, emphasising trajectory-tracking control capability under changing disturbance and payload conditions. Part 5 - provides comprehensive simulation results, comparing the performance of the proposed improved LQR controller with that of the classical PID/LQR controller across various flight trajectories and scenarios, using the integral error metrics (ISE, ITSE, ITAE) and statistical indices (MSE, RMSE) for quantitative assessment. The quadcopter UAV hardware was then deployed, and flight experiments were conducted under wind disturbances and varying loads using the proposed algorithm. The final part is the conclusion and references.

2. RELATED WORKS

Research from [24-29] has focused on applying LQR control algorithms to improve system performance and optimize control capabilities for stable and responsive complex objects. LQR has been shown to be an effective tool in minimizing errors and enhancing robustness. This algorithm also allows optimizing energy usage, minimizing oscillations and improving system accuracy under various operating conditions. Research [24] proposed LQR to solve the complete tracking problems in the identification and control of aircraft systems even with moderate measurement deviations and noise as long as the aircraft model is accurately determined. Research [25] proposed a dual-loop LQR controller for autonomous UAVs, with the inner loop using LQR for stabilization and the outer loop using Fuzzy-PID to track reference signals. In [26], the authors proposed LQR for a small UAV, optimized the unmanned system, and conducted simulation experiments. In study [27], the authors applied LQR to reduce errors and overshoot, helping the aircraft maintain excellent stability and accuracy. In [28], the authors presented a discrete LQR controller combined with integral action to stabilize the attitude and altitude of a four-rotor helicopter, thereby reducing energy consumption and achieving the desired UAV behavior. In study [29], the authors compared LQR, LQG, and Kalman filter controllers to control the pitch angle of a fixed-wing UAV. However, the aforementioned studies often rely on specific adaptive law structures or require additional modeling, and they do not directly optimize control performance based on quantitative metrics. Therefore, there remains a research gap in developing a systematic LQR weighting adjustment method that is directly based on trajectory tracking performance and can be flexibly applied under nonlinear conditions, with disturbances and varying loads.

3. ORIGINALITY

The study is demonstrated through three main contributions: (i) the integration of a complete nonlinear 6-DOF dynamic model for the UAV quadcopter, accounting for the effects of disturbance and payload; (ii) propose an iterative LQR control method, in which the weight matrices Q and R are determined by an iterative optimization algorithm, to minimize the weighted average control quality index of the ITAE, ISE, RMSE criteria; (iii) the assessment of control performance across various trajectory types.

4. SYSTEM DESIGN

4.1. UAV Quadcopter Dynamic Model

In this study, we assume that the structure of the quadcopter is rigid and symmetrical, with the center of mass located at the origin of the body frame. The propellers are also considered rigid, with the thrust and drag generated by them proportional to the square of the rotational speed, and the motors are classified and numbered from 1 to 4 (Figure 1). Each motor plays a crucial role in controlling the movements of the quadcopter, allowing it to perform actions such as ascending, descending, and rotating. Fig.1 illustrates the structure and the forces acting on the quadcopter in three-dimensional space, where the four propellers are securely mounted on the arms. Each propeller generates an upward thrust along with a torque around its axis, with corresponding angular velocities, helping to adjust the direction and speed of the quadcopter. The coordinate system (x_B, y_B, z_B) is attached to the quadcopter. In contrast, the coordinate system (x,y,z) represents two UAV different movements, facilitating the adjustment of thrust to perform movements such as ascending, descending, rotating, and lateral. The quad-

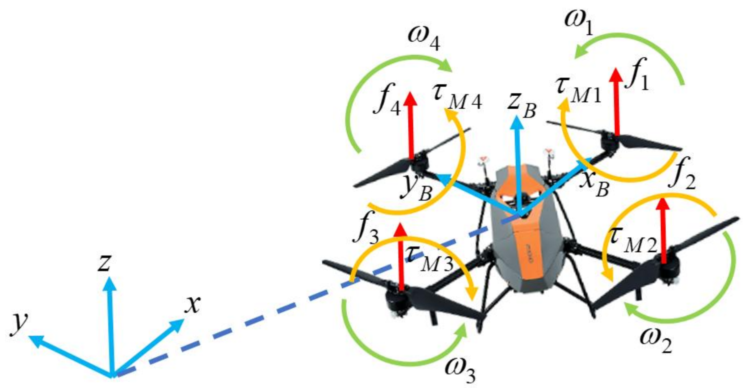


Figure 1. Inertial reference frame and body reference frame of UAV quadcopter

copter, an unmanned aerial vehicle (UAV), uses four independently operating propellers, allowing precise movement control by adjusting each propeller's speed.

The Newton-Euler equations provide a comprehensive model of the dynamics of the quadcopter, assuming it is a rigid body. For the translational component, the UAV is influenced by two main forces: the net thrust $[0,0,F]^T$ and the gravitational force $[0,0,mg]^T$, where m is the mass of the quadcopter and g is the acceleration due to gravity. Conversely, the rotational dynamics are governed by the change in angular momentum, determined by the difference between the external torque $\left[\tau_\phi,\tau_\theta,\tau_\psi\right]^T$ and the restoring force ω_δ. Therefore, the nonlinear dynamics of the quadcopter are described here through the Euler angles ϕ,θ and ψ, which defines its orientation in three-dimensional space [2,5,6],

$$\begin{aligned}\ddot{x} &= \frac{F}{m}(cos\,\psi\, sin\,\theta\, cos\,\phi + sin\,\psi\, sin\,\phi)\\ \ddot{y} &= \frac{F}{m}(sin\,\psi\, sin\,\theta\, cos\,\phi - cos\,\psi\, sin\,\phi)\\ \ddot{z} &= \frac{F}{m}(cos\,\phi\, cos\,\theta) - g\end{aligned} \tag{1}$$

$$\begin{bmatrix}\dot{p}\\ \dot{q}\\ \dot{r}\end{bmatrix} = \begin{bmatrix}(I_{yy}-I_{zz})qr/I_{xx}\\ (I_{zz}-I_{xx})pr/I_{yy}\\ (I_{xx}-I_{yy})pq/I_{zz}\end{bmatrix} - I_M\begin{bmatrix}q/I_{xx}\\ -p/I_{yy}\\ 0\end{bmatrix}\omega_\delta + \begin{bmatrix}\tau_\phi/I_{xx}\\ \tau_\theta/I_{yy}\\ \tau_\Psi/I_{zz}\end{bmatrix} \tag{2}$$

$$\begin{aligned}\dot{p} = \ddot{\phi} &= \left(\frac{I_{yy}-I_{zz}}{I_{xx}}\right)\dot{\theta}\dot{\psi} - \left(\frac{I_M\omega_\delta}{I_{xx}}\right)\dot{\theta} + \left(\frac{\tau_\phi}{I_{xx}}\right)\\ \dot{q} = \ddot{\theta} &= \left(\frac{I_{zz}-I_{xx}}{I_{yy}}\right)\dot{\psi}\dot{\phi} + \left(\frac{I_M\omega_\delta}{I_{yy}}\right)\dot{\phi} + \left(\frac{\tau_\theta}{I_{yy}}\right)\\ \dot{r} = \ddot{\psi} &= \left(\frac{I_{xx}-I_{yy}}{I_{zz}}\right)\dot{\theta}\dot{\phi} + \left(\frac{\tau_\psi}{I_{zz}}\right)\end{aligned} \tag{3}$$

where, $\omega_\delta = \omega_1 - \omega_2 + \omega_3 - \omega_4$ is the total speed of the propellers, m is the mass of the quadrotor, I_M is the moment of inertia of the rotor. I_{xx}, I_{yy}, and I_{zz} are the moments of inertia of the quadrotor along the *x,y,z* axes. The angular velocity vector $[p, q, r]^T$ is related to the Euler angles through the following transformation, as detailed in [2,5,6].

$$\begin{bmatrix} p \\ q \\ r \end{bmatrix} = \begin{bmatrix} 1 & 0 & -Sin\,\theta \\ 0 & Cos\,\phi & Cos\,\theta\, Sin\,\phi \\ 0 & -Sin\,\phi & Cos\,\theta\, Cos\,\phi \end{bmatrix} \begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} \tag{4}$$

The inputs can be defined as follows [2, 4-6],

$$\begin{aligned} F &= U_1 = k(\omega_1^2 + \omega_2^2 + \omega_3^2 + \omega_4^2) \\ \tau_\phi &= U_2 = l\,k\,(\omega_4^2 - \omega_2^2) \\ \tau_\theta &= U_3 = l\,k\,(\omega_3^2 - \omega_1^2) \\ \tau_\psi &= U_4 = b(\omega_1^2 - \omega_2^2 + \omega_3^2 - \omega_4^2) \end{aligned} \tag{5}$$

where, the input signal U_1 is the total thrust of the four rotors, while U_2, U_3 and U_4 represent the moments for roll, pitch, and yaw, respectively. The lift force coefficient is denoted by k, the rotor's torque coefficient by b and the distance between the propeller and the UAV's center of mass by l.

4.2. The Iterative LQR Control Design For UAV Quadcopter

This article proposes an iterative LQR controller system to stabilize the UAV quadcopter trajectory tracking control in the disturbance environment, using the iterative optimization algorithm to determine the Q and R weight matrices to minimize the average control quality index of ITAE, ISE, RMSE criteria.

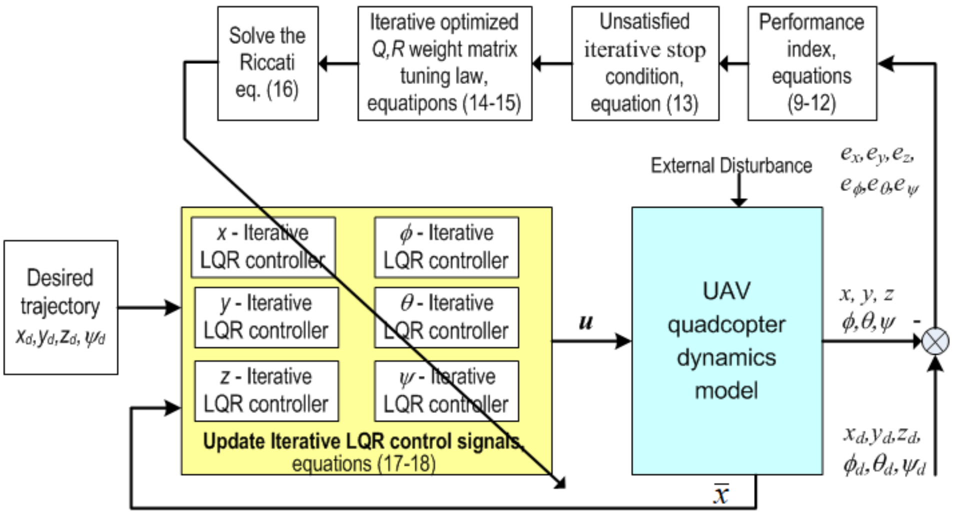


Figure 2. Block diagram of the iterative LQR control system for UAV quadcopter

Figure 2 presents the control diagram of the system, where the LQR controllers are assigned to each state variable: position x,y, altitude z, and the roll angle θ, pitch angle ϕ, and yaw angle ψ. This system compares the desired input value for each state variable with the actual measured value, generating a deviation signal. This signal is then fed into the corresponding iterative LQR controllers, which calculate the necessary control feedback to minimize the error. Additionally, the control signals from the x and y position controllers are processed through the reverse solution block to compute the desired roll angles, assisting the UAV in maintaining a stable position in three-dimensional space. With the external disturbances block simulating external disturbances, this control system still performs well, helping the quadcopter closely follow the desired trajectory and ensuring stability under complex conditions.

The cost minimization function of iterative LQR control in this study, including state error and control effort, is defined as follows [27, 30],

$$J = \int_0^{\infty} (\bar{x}^T Q \bar{x} + u^T R u)\, dt \tag{7}$$

where Q and R is weighting matrices determining the relative importance of state error and control input. Q is configured to prioritize position tracking and adjust control effort, while R is tuned to balance control smoothness. These weighting matrices play a crucial role in assessing the relative importance of current errors as well as the energy consumption of the system. Choosing appropriate values for the weighting matrices Q, R directly affects the control system performance and mainly based on experience. If the Q matrix is set too high, the system may overreact to minor deviations, leading to unnecessary control actions that could cause instability. Conversely, if R is too low, the system may prioritize minimizing control effort over achieving the desired state performance, resulting in slow response times.

To improve the UAV quadcopter system trajectory tracking control quality, this study proposes a new iterative optimization algorithm to minimize the objective function based on the control error, as follows:

Step 1: Initialize the system with Q, R as the unit matrix

$$Q = I_n, R = I_m \tag{8}$$

where, n is the number of states, and m is the number of control signals.

Step 2: Calculate the error indices for the iteration k-th.

Let PI_k is perforance index, total error of the k-th iteration, which is also the iterative optimization objective function, obtained from the weighted average of the *ITAE, ISE, RMSE* indices, defined as follows:

$$PI_k = \alpha \times ITAE_k + \beta \times ISE_k + \gamma \times RMSE_k \tag{9}$$

The *ITAE* (Integral of Time-weighted Absolute Error) measures the cumulative absolute error, with a weighting factor that increases with time. It combines the heavy penalization of large errors with the emphasis on reducing long-lasting errors. ITAE is represented by the formula

$$ITAE = \int_0^\infty t|e(t)|\,dt \tag{10}$$

The *ISE* (Integral of Squared Error) measures the cumulative square of the error over time. It penalizes large errors more heavily than small errors. This is useful when it's critical to minimize large deviations from the desired output. ISE is represented by the formula

$$ISE = \int_0^\infty e^2(t)\,dt \tag{11}$$

The *RMSE* provides a measure of the average magnitude of the error, expressed in the same units as the output variable, making it easier to interpret. Essentially, *RMSE* provides a good idea of how much the control system output deviates from the desired setpoint on average. *RMSE* (Root Mean Squared Error) is sensitive to outliers due to the square of the error terms. *RMSE* is calculated as

$$RMSE = \sqrt{\frac{1}{T}\int_0^T e^2(t)dt}\,;\ T - \text{calculated time period} \tag{12}$$

In this UAV quadcopter trajectory tracking control study, the highest priority is given to ITAE because it reflects long-term oscillations with α=0.5, *ISE* for instantaneous error with β=0.3 and RMSE reflects the degree of actual average deviation with γ=0.2

Step 3: Check satisfied termination

The iterative algorithm will continue as long as the total error of the k-th iteration is greater than the predefined threshold:

$$PI_k > \delta \tag{13}$$

where, δ is the convergence threshold.

Step 4: Calculate Q_{ii} using the logarithmic normalization error formula.

The diagonal elements of Q_{ii} are updated based on the logarithmic transformation of the normalized error:

$$Q_{ii}^{(k+1)} = log\left(1 + \frac{e_i^{(k)}}{\varepsilon + min_j\, e_j^{(k)}}\right) \tag{14}$$

where ε is a very small constant to avoid division by 0, and k is the current iteration.

Step 5: Calculate R_{jj} based on the actual control signal $u_j^{(k)}(t)$

The diagonal elements of R_{jj} are updated based on the average magnitude of the control signal $u_j^{(k)}(t)$

$$R_{jj}^{(k+1)} = log\left(1 + \frac{1}{T}\int_{0}^{T}\left|u_j^{(k)}(t)\right|dt\right) \tag{15}$$

where, T is the simulation calculation time.

Step 6: Update the LQR control with the new Q,R.

Recalculate the gain matrix K with the new weighting matrices $Q^{(k+1)}$, $R^{(k+1)}$ and update control signals.

After obtaining the above matrices Q,R, solved the Riccati algebra equation to determine the symmetric matrix P, and then determined the control gain matrix K, and finally determined the improved optimal control vector u.

$$A^T P + PA - PBR^{-1}B^T P + Q = 0 \tag{16}$$

$$K = R^{-1}B^T P \tag{17}$$

$$u = -K\,\bar{x} \tag{18}$$

5. EXPERIMENT AND ANALYSIS

The simulation of the UAV quadcopter trajectory tracking control system using the iterative LQR control algorithm is implemented in the MATLAB to evaluate the effectiveness of the proposed controller in maintaining stability and following the desired trajectory, as described at Table 1.

Table 1. The parameters of the UAV quadcopter

Parameter	Symbol	Value
Quadcopter mass	m	0.468 kg
Arm length	l	0.225 m
Gravity	g	$9.81 m/s^2$
Inertia moment of the rotor	I_M	$3.357e-5 kg.m^2$
Thrust factor of rotor	k	$2.980e-6 N.s^2$
Drag coefficient	b	$1.140e-7 N.m.s^2$
Inertial constants	I_{xx}, I_{yy}	$4.856e-3 kg.m^2$
Inertial constants	I_{zz}	$8.801e-3 kg.m^2$

The study conducts a comprehensive analysis of the control performance of the iterative LQR controller compared to traditional LQR/PID controller through three characteristic operational scenarios: (i) the ideal conditions without disturbance, (ii) the external disturbance environment, (iii) the combination of disturbance and varying payload. Each scenario is tested with three complex different flight trajectory types. The control performance index is evaluated through time-integrated error indices (ISE, ITAE) and statistical indices (MSE, RMSE), providing a quantitative and objective insight into the system accuracy, responsiveness, and stability under each operating condition.

5.1. Evaluation of the quality of UAV quadcopter control system with iterative LQR control and traditional LQR control

The performance evaluation results of the UAV quadcopter trajectory tracking control system with iterative LQR control and traditional LQR control are presented in Table 2 and Figure 3.

Table 2. Comparison of UAV quadcopter control quality with the fixed chosen Q,R (traditional LQR) and iterative optimal Q,R (iterative LQR)

Quality Index / Controller	ITAE_ Fixed	ITAE_ Opt	ISE_ Fixed	ISE_ Opt	RMSE_ Fixed	RMSE_Opt	e_Fixed	e_Opt
x	640.7569	623.8496	0.6323	0.5983	0.0562	0.0547	384.6494	374.4947
y	567.4650	552.6577	3.6335	3.4709	0.1348	0.1317	341.5825	332.6490
z	128.4262	51.1978	0.2434	0.1703	0.0349	0.0292	77.1322	30.7727
ϕ	62.4495	62.3231	1.2665	1.1935	0.0796	0.0773	37.8576	37.7597
θ	64.7210	64.6718	0.1445	0.1366	0.0269	0.0261	38.8786	38.8467
ψ	21.7146	5.4867	0.0003	0.0000	0.0012	0.0003	13.0290	3.2920

The results show that the iterative LQR controllers with the Q,R iterative optimized weight matrices improves the control performance on all quality metrics (ITAE, ISE, RMSE). Notably, the ITAE value on the z-axis is significantly reduced from 128.43 to 51.20, corresponding to an improvement of more than 60%, reflecting a significant reduction in system delay. Similarly, the yaw angle ψ shows a marked improvement, with the ITAE decreasing from 21.71 to 5.49 and the RMSE from 0.0012 to 0.0003, indicating a significant improvement in directional stability. In addition, the total errors each controller are also significally improved, such as with the z-axis decreasing from 77.13 to 30.77 and ψ-angle yaw decreasing from 13.03 to 3.29. These results confirm that the automatic optimization of Q-R weights provides superior performance in UAV control, especially in adjusting the altitude and direction of flight, thus improving the trajectory tracking accuracy and system stability under changing dynamic conditions.

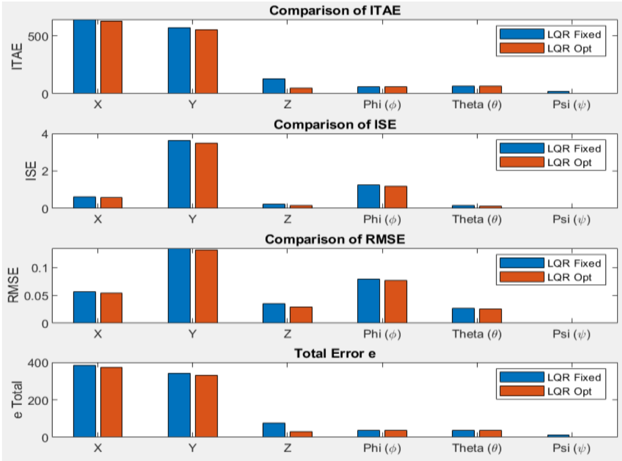
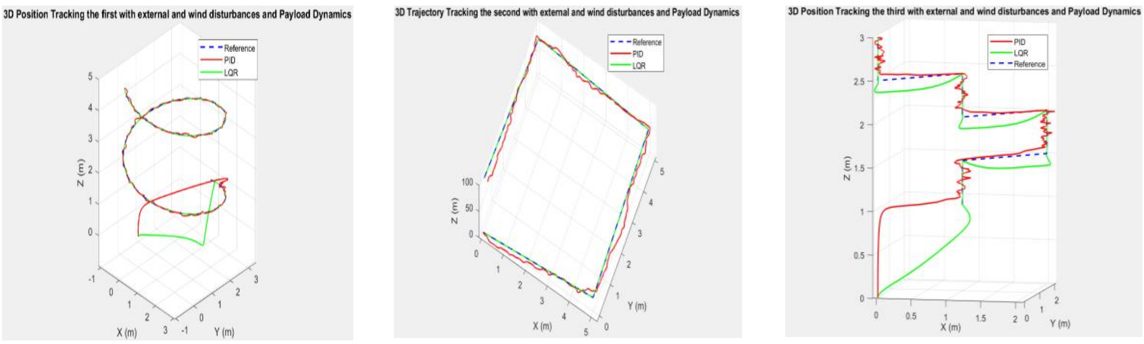


Figure 3. Comparison chart of UAV quadcopter control quality with the fixed Q,R (traditional LQR) and iterative optimal Q,R

5.2. Evaluation of the quality of UAV quadcopter control system with iterative LQR control and traditional PID control

Next, the control performance comparison between the traditional PID controller and the iterative LQR controller is conducted for the 6-DOF UAV quadcopter on three different trajectories under disturbance and payload variation.

Figures 4,5,6,7 illustrate the response of the iterative LQR UAV quadcopter control system to three different trajectories under disturbance and variable payload, showing a clear difference in performance between the conventional PID controller and the iterative LQR controller. In each figure, it can be seen that the iterative LQR controller always maintains the trajectory close to the reference line, with smaller oscillation amplitude and shorter response time. Especially on the *x, y* position axes and the phi, theta, psi rotation angles, the iterative LQR signal shows better stability and noise rejection than the conventional PID. Meanwhile, the PID controller tends to cause oscillation around the equilibrium point, noticeable overshoot and larger hysteresis, especially when the system is subjected to disturbance or sudden load changes. The response curve of the PID shows significant instability in the third trajectory (Fig.7), while the iterative LQR still maintains smoothness and accuracy. Analysis of these response curves shows that the iterative LQR is a more suitable controller for systems requiring high accuracy, good tolerance to disturbances, and long-term stability. In contrast, PID, although capable of faster response in some short-term situations, is generally less efficient and more susceptible to disturbances.



(a) (b) (c)

Figure 4. The UAV quadcopter 3D position simulation with variable payload and disturbance of the iterative LQR and traditional PID controllers. a) First trajectory, b) Second trajectory, c) Third trajectory

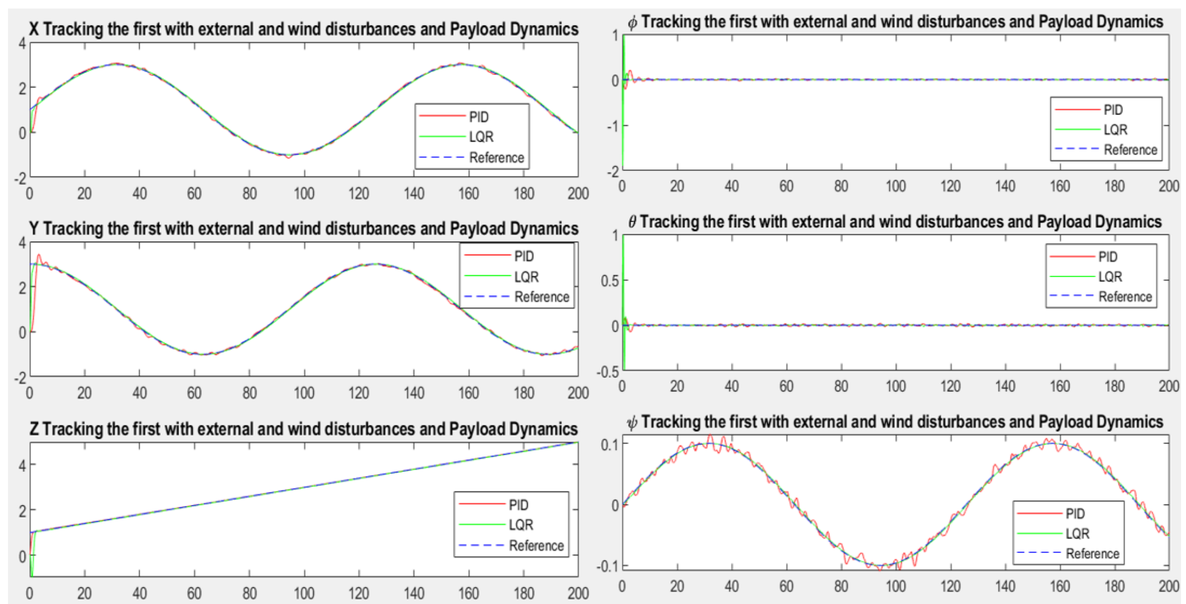


Figure 5. Time response of position and attitude states (*x [m], y [m], z [m], φ [rad], θ [rad], ψ [rad]*) under disturbance and payload variation for the first trajectory of the iterative LQR and traditional PID controllers.

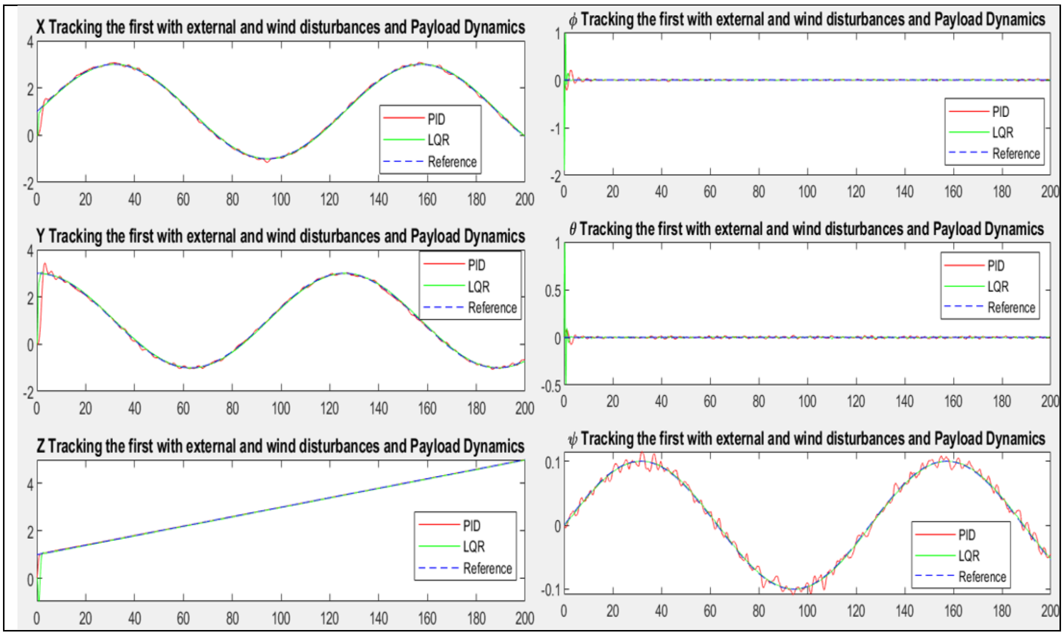


Figure 6. Time response of position and attitude states (*x [m], y [m], z [m], φ [rad], θ [rad], ψ [rad])* under disturbance and payload variation for the second trajectory of the iterative LQR and traditional PID controllers

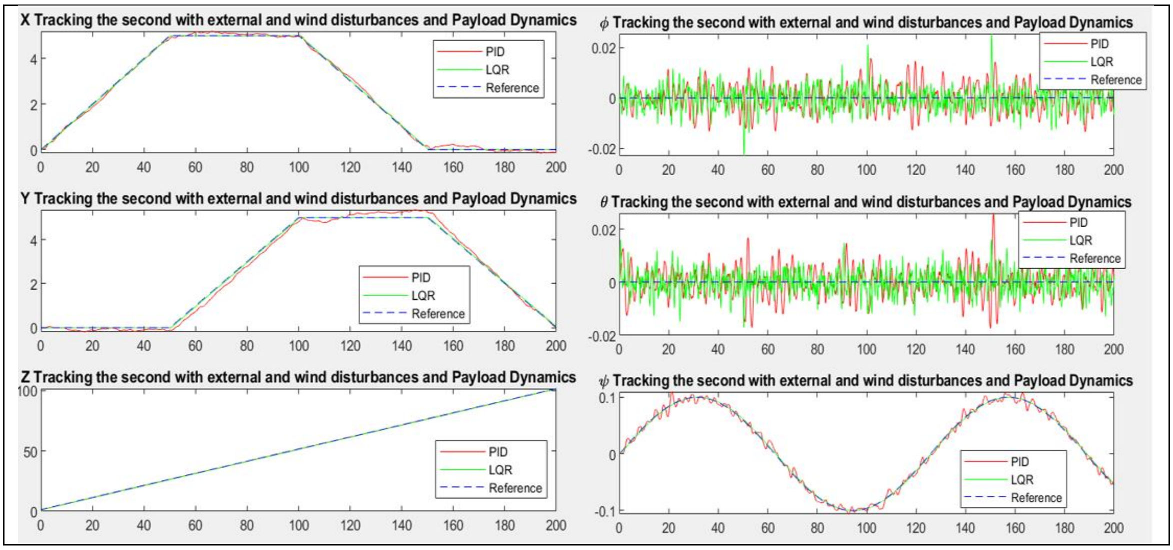


Figure 7. Time response of position and attitude states (*x [m], y [m], z [m], φ [rad], θ [rad], ψ [rad])* under disturbance and payload variation for the third trajectory of the iterative LQR and traditional PID controllers.

The three control performance comparison tables between the PID controller and the iterative LQR controller (Tables 3,4,5) show that the iterative LQR generally outperforms the PID in most of the measurements, especially in terms of errors along the position axis. Specifically, in Table 3, for the first trajectory under the influence of noise and load, the iterative LQR shows significantly lower mean square error (MSE) and root mean square error (RMSE) than the PID on most axes, notably 0.0677). Similar results are observed in Table 4 (second trajectory), where the iterative LQR continues to show advantages on the *x,y*-axes, with an MSE of only 0.0011, while the PID has 0.0263 and 0.0150, respectively. However, it is worth noting that on the *z*-axis, the PID performs better with an MSE of 0.0210 compared to 0.0672 for the iterative LQR, indicating that factors may affect the performance of the LQR on this axis in the second trajectory. In Table 5, with the third trajectory, the iterative LQR still shows superiority on

Table 3. The control quality evaluation between the iterative LQR and PID controllers with the first trajectory with variable payload-disturbance

Quality Index / Controller	MES_PID	MES_ improved LQR	RMES_PID	RMES_ improved LQR
x	0.0099782	0.0040539	0.099891	0.06367
y	0.067748	0.017226	0.26028	0.13125
z	0.0016589	0.017421	0.04073	0.13199
ϕ	0.00048717	0.0046679	0.022072	0.068322
θ	0.00010706	0.0016485	0.010347	0.040602
ψ	3.9607e-05	2.5716e-06	0.0062934	0.0016036

Table 4. The control quality evaluation between the iterative LQR and PID controllers with the second trajectory with variable payload-disturbance

Quality Index / Controller	MES_PID	MES_ improved LQR	RMES_PID	RMES_ improved LQR
x	0.0263	0.0011	0.1622	0.0337
y	0.0150	0.0011	0.1224	0.0337
z	0.0210	0.0672	0.1449	0.2593
ϕ	0.0000	0.0000	0.0053	0.0045
θ	0.0000	0.0000	0.0059	0.0043
ψ	0.0000	0.0000	0.0061	0.0016

Table 5. The control quality evaluation between the iterative LQR and PID controllers with the third trajectory with variable payload-disturbance

Quality Index / Controller	MES_PID	MES_ improved LQR	RMES_PID	RMES_ improved LQR
x	0.0496	0.0077	0.2228	0.0876
y	0.0401	0.0070	0.2003	0.0839
z	0.0016	0.0026	0.0405	0.0507
ϕ	0.000063	0.002699	0.007944	0.051954
θ	0.000067	0.003205	0.008211	0.056613
ψ	0.000045	0.000003	0.006733	0.001669

From the entire analysis and simulation data presented in the article, it can be concluded that the iterative LQR controller demonstrates superior performance compared to PID controller in terms of stability and accuracy in controlling the 6-DOF UAV quadcopter, primarily when the system operates under complex conditions such as variable payload and disturbance. Specifically, through the data tables (Tables 2–5) and simulation images (Figures 4–7), iterative LQR controller consistently achieves lower MSE and RMSE metrics on the position x,y-axes, reflecting its ability to maintain a smooth trajectory and minimize oscillations. Although PID controller may perform better in some rotational components, such as phi and theta, in simpler trajectories or on the z-axis, but iterative LQR controller shows exceptional stability as complexity increases, as evidenced in the third trajectory. Moreover, PID controller exhibits significant oscillations in variable payload and disturbance conditions, particularly at yaw angles (ψ) and in states requiring quick responses. At the same time, iterative LQR controller maintains smooth feedback and noticeably more minor errors, demonstrating good noise resistance. These results prove that the iterative LQR controller is optimal for systems requiring precise control, long-term stability, and adaptability to external fluctuations. At the same time, despite its simplicity and ease of implementation, the PID controller is limited in scalability under complex and nonlinear conditions. Table 6 presents the performance index of the PID and iterative LQR controllers under disturbance and varying payload.

Table 6. The performance index of the iterative LQR and PID controllers

Evaluation Criteria	Iterative LQR Controller Performance	PID Controller Performance
Trajectory Tracking Accuracy	High (RMSE is always lower)	Good in ideal conditions, poor with disturbances
Disturbance Rejection Capability	Stable and quick respond	Very prone to oscillation
Payload Adaptation Capability	Stable under varying payload	Performance degradation
Euler Angle Control (ϕ,θ,ψ)	Stable, can be improved with better Q optimization	Responsive in light conditions, strong oscillations under disturbance
Energy Consumption (indirect)	More efficient due to optimized control signals	More fluctuations lead to higher control costs

5.3. Experimental Validation and Hardware Implementation

To verify the feasibility of the proposed control method, an experimental quadcopter UAV platform was developed based on the hardware platform illustrated in Figure 8. The system includes a quadcopter frame, four BLDC motors, electronic speed controllers (ESCs), the central flight controller, and IMU sensors. The flight controller is implemented on an Arduino Nano microcontroller. The main task is to stabilize the attitude and follow the trajectory in real-time. IMU data is used to estimate the angular state and angular velocity. These signals are fed into feedback control loops. The propulsion system generates thrust and torque to control 6-DOF UAV quadcopter movement. Figure 9 describes the actual UAV quadcopter used in this experiment.

The experiment was set up to evaluate the performance of the PID controller. The experiments were conducted outdoors. The reference trajectories were pre-designed. The PID parameters were tuned empirically. The goal was to ensure stability and acceptable responsiveness. The evaluation criteria included trajectory tracking accuracy, attitude stability, and disturbance resistance.

The experimental results system achieved an acceptable level of performance across various operations. In a windless environment, the UAV maintained good stability and position hold. However, when wind disturbances occurred, the system tended to drift in the direction of the environmental influence, indicating limited disturbance resistance. In terms of control capability, the UAV could move in basic directions and operate at various speed levels, meeting manual control requirements. The flight time was approximately the system about 30-90 minutes with a weight of about 0-800grams, this value depends on battery type conditions and the actual operational conditions.

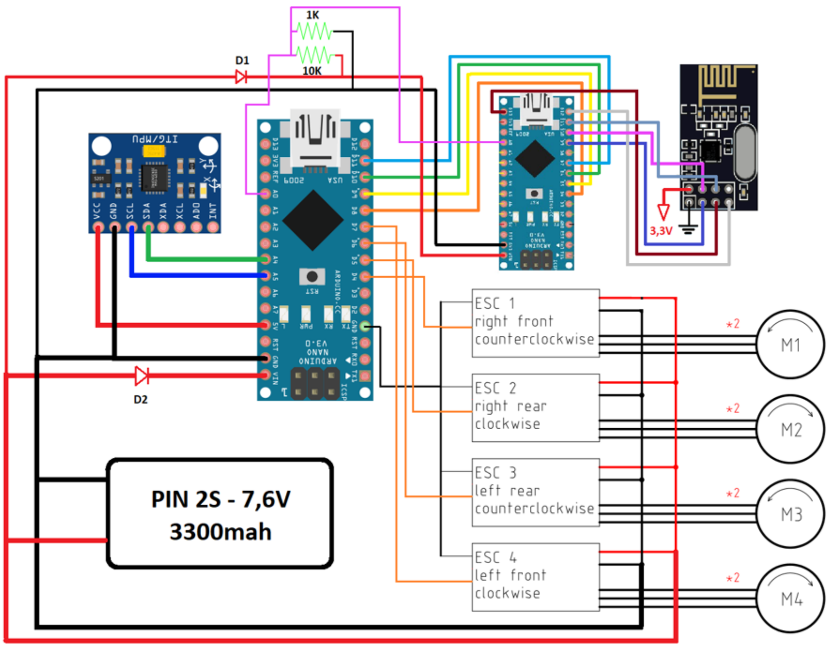


Figure 8. Experimental quadcopter UAV control hardware platform.

Figure 9. Actual UAV quadcopter for experiment

Regarding control range, under ideal environmental conditions (low interference and no obstacles), the control distance can exceed 200 m. Conversely, in environments with interference or many obstacles, this range decreases to about 100 m or lower, depending on the level of interference.

The iterative LQR controller was then implemented independently on this quadcopter UAV. The experimental results under low wind disturbance and varying payload conditions showed that the UAV's flight attitude was maintained more stably and operational errors were significantly reduced.

Figure 10. Experimental UAV flight under disturbance and payload variation.

The experimental flight data of the UAV quadcopter show that the flight altitude remained stable at the prescribed setpoints of 2 m and 4 m, as shown at Figure 11a. The roll angle was maintained near 0 during wind disturbances, indicating that the UAV maintained good flight stability, as shown at Figure 11b. The pitch angle was automatically adjusted to an appropriate value to compensate for wind-induced drift, thereby ensuring stable flight performance, as shown at Figure 11c. The yaw angle remained stable throughout the experiment, indicating that the UAV successfully maintained its heading during flight, as shown at Figure 11d.

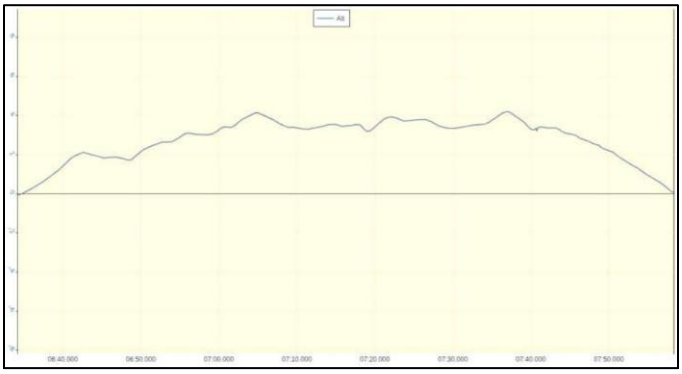


Figure 11a. Experimental flight attitude response under conditions of low wind disturbance and payload

Figure 11b. Experimental roll angle response under conditions of low wind disturbance and payload

Figure 11c. Experimental pitch angle response under conditions of low wind disturbance and payload

Figure 11d. Experimental yaw angle response under conditions of low wind disturbance and payload

6. CONCLUSION

This article presents an iterative LQR control method based on the application of iterative optimization algorithm to optimally adjust the weight matrix Q, R to improve the trajectory tracking control quality for 6-DOF UAV quadcopter. The objective function used in this study is based on the weighted combination of three quality indicators ITAE, ISE, RMSE. Specifically, the Q matrix is updated through the logarithmic transformation of the normalized error indices ITAE, ISE, RMSE. The R matrix is updated based on the average magnitude of the control signal over time. This performance-based weight adjustment method significantly improves the trajectory tracking ability and sustainability of the iterative LQR controller under the conditions of noise and load changes. Simulation results confirm that the improved adaptive LQR controller outperforms the conventional LQR and the classical PID controller in terms of accuracy, stability and disturbance rejection. The UAV quadcopter trajectory tracking control system under the impact of disturbance and variable payload, when using the iterative LQR controller, is more efficient than the traditional LQR/PID controller from 6.6% to 41.8%, especially the superior efficiency in the x,y,ψ control loops. Preliminary experimental results on a quadcopter UAV hardware contribute to clarifying the effectiveness of the iterative LQR control algorithm proposed in this paper, demonstrating its potential for practical application.

In the future, the research team will focus on developing advanced solutions such as optimizing the Q,R weight matrices using evolutionary algorithms or reinforcement learning, converting to a quaternion error model to improve the accuracy of attitude control, and integrating a Kalman filter to enhance state estimation under measurement noise conditions.

REFERENCES

[1] Do H.T., Trinh L.M., et al., **AR Marker Detection Based Autonomous Attitude Control for an Indoor Non-GPS Aided Quadcopter**, *Advances in Engineering Research and Application, ICERA 2023*, Spinger, 228–238,

2024. https://doi.org/10.1007/978-3-031-62238-0_26

[2] Van, An Vo and Duy, Hung Ha, **Control Strategies for 6-DOF Quadcopter UAVs: Cascade PID Stabilization in White Noise Conditions**, *Makara Journal of Technology,* 29, 119-132, 2025. https://doi.org/10.7454/mst.v29i3.1697

[3] Mien T.L., Tu T.N., **Design and Quality Evaluation of the Position and Attitude Control System for 6-DOF UAV Quadcopter Using Heuristic PID Tuning Methods**, *International Journal of Robotics and Control Systems*, 4(4), 1712-1730, 2024. https://doi.org/10.31763/ijrcs.v4i4.1594

[4] S. Abdelhaya, A. Zakriti, **Modeling of a Quadcopter Trajectory Tracking System Using PID controller,** *Procedia Manufacturing*, 32, 564-571, 2019. https://doi.org/10.1016/j.promfg.2019.02.253.

[5] Vo Van An, Trinh Luong Mien, Nguyen Van Binh, **A comprehensive survey of UAV control algorithms: Integrating classical methods with artificial intelligence for enhanced trajectory tracking**, *AIMS Electronics and Electrical Engineering*, 10, 26-53, 2026. https://doi.org/10.3934/electreng.2026002

[6] Trinh L.M., Tran N.T., Vo V.A. **Cascade PID Control for Altitude and Angular Position Stabilization of 6-DOF UAV Quadcopter**, *International Journal of Robotics and Control Systems*, 4 (2), 814-831, 2024. https://doi.org/10.31763/ijrcs.v4i2.1410

[7] Trinh L.M, Vo V.A**, Modeling and control of 6-DOF UAV quadcopter using PID controllers arcording to Ziegler-Nichols**, *III International Scientific and Practical Conference Intelligent Transport Systems*, Moscow, 438-492, 2024. https://doi.org/10.30932/9785002446094-2024-483-492

[8] Tamami, N., Pitowarno, E., & Astawa, I. G. P. **A Combination of PD Controller and PIAFC for Stabilization of X Configuration Quadcopter**. *EMITTER International Journal of Engineering Technology*, 2(1), 1-16, 2014. https://doi.org/10.24003/emitter.v2i1.13

[9] V. Nekoukar and N. M. Dehkordi, **Robust path tracking of a quadrotor using adaptive fuzzy terminal sliding mode control**, *Control Engineering Practice*, 110, 104763, 2021. https://doi.org/10.1016/j.conengprac.2021.104763

[10] H. Housny, E. A. Chater and H. El Fadil, **New Deterministic Optimization Algorithm for Fuzzy Control Tuning Design of a Quadrotor**, *2019 5th International Conference on Optimization and Applications*, 1-6, 2019. https://doi.org/10.1109/ICOA.2019.8727622

[11] C. Ha, Z. Zuo, F. B. Choi, D. Lee. **Passivity-based adaptive backstepping control of quadrotor-type UAVs**, *Robotics and Autonomous Systems*, 62(9), 1305-1315, 2014. https://doi.org/10.1016/j.robot.2014.03.019.

[12] G. Sheng and G. Gao, **Research on the Attitude Control of Civil Quad-Rotor UAV Based on Fuzzy PID Control**, *2019 Chinese Control And Decision Conference*, 4566-4569, 2019. https://doi.org/10.1109/CCDC.2019.8832855

[13] M. Jiang. **Application of Fuzzy PID Control in UAV Control System**, *2021 Third Int. Conf. on Inventive Research in Computing Applications*, 197-200, 2021. https://doi.org/10.1109/ICIRCA51532.2021.9544986

[14] Vo Van An. **Leader-follower formation control of quadcopter UAVs using a Mamdani fuzzy PID controller with performance evaluation under disturbance conditions**, *AIMS Electronics and Electrical Engineering*, 10(3), 473-503, 2026. https://doi.org/10.3934/electreng.2026019

[15] C. E. Luis, M. Vukosavljev and A. P. Schoellig. **Online Trajectory Generation With Distributed Model Predictive Control for Multi-Robot Motion Planning**, *IEEE Robotics and Automation Letters*, 5(2), 604-611, 2020. https://doi.org/10.1109/LRA.2020.2964159

[16] S. Elkhatem and S. N. Engin. **Robust LQR and LQR-PI control strategies based on adaptive weighting matrix selection for a UAV position and attitude tracking control**, *Alexandria Engineering Journal*, 61(8), 6275-6292, 2022. https://doi.org/10.1016/j.aej.2021.11.057

[17] M. Karahan, M. Inala, C. Kasnakoglu. **Fault Tolerant Super Twisting Sliding Mode Control of a Quadrotor UAV Using Control Allocation**, *International Journal of Robotics and Control Systems*, 3(2), 270-285, 2023. https://doi.org/10.31763/ijrcs.v3i2.994

[18] Nguyen V.C., Trinh L.M., et al**. Trajectory Tracking Control for a Quadcopter under External Disturbances**, *Engineering, Technology & Applied Science Research*, 14(6), 17620-17628, 2024. https://doi.org/10.48084/etasr.8449

[19] S. Yang and B. Xian. **Exponential Regulation Control of a Quadrotor Unmanned Aerial Vehicle With a Suspended Payload**, *IEEE Transactions on Control Systems Technology*, 28(6), 2762-2769, 2020. https://doi.org/10.1109/TCST.2019.2952826

[20] G.V. Raffo, M.G. Ortega, F.R. Rubio. **An integral predictive/nonlinear H∞ control structure for a quadrotor helicopter**, *Automatica*, 46(1), 29-39, 2010. https://doi.org/10.1109/TCST.2019.2952826

[21] E. N. Mobarez, A. Sarhan and M. Ashry. **Fractional order PID Based on a Single Artificial Neural Network Algorithm for Fixed wing UAVs**, *2019 15th International Computer Engineering Conference*, 168-174, 2019. https://doi.org/10.1109/ICENCO48310.2019.9027378

[22] N. H. Sahrir, A. M. Basri. **Radial Basis Function Network Based Self-Adaptive PID Controller for Quadcopter**, *International Journal of Robotics and Control Systems*, 4(1), 151-173, 2019. https://doi.org/10.31763/ijrcs.v4i1.1261

[23] Ma'arif, I. Suwarno, E. Nur'aini, N. M. Raharja. **Altitude Control of UAV Quadrotor Using PID and Integral State Feedback**, *BIO Web of Conferences*, 65, 7011, 2023. https://doi.org/10.1051/bioconf/20236507011

[24] Franciszek Dul, Piotr Lichota and Artur Rusowicz. **Generalized Linear Quadratic Control for a Full Tracking Problem in Aviation**, *MDPI (sensor)*, 20(10), 2955, 2020. https://doi.org/10.3390/s20102955

[25] Malik M. A. Al-Isawi, Adnan J. Attiya, Julius O. Adoghe. **UAV Control Based on Dual LQR and Fuzzy-PID Controller**, *Al-Khwarizmi Engineering Journal*, 16(3), 43-53, 2020. https://doi.org/10.22153/kej.2020.08.001

[26] Hteik Tin Cho Nyunt, Hla Myo Tun, Thanda Win, **Development of LQR Controller Design for Small Unmanned Aerial Vehohicle**, *Research World International Conference*, Singapore. 2018

[27] Tri Kuntoro Priyambodo, Oktaf Agni Dhewa and Try Susanto, **Model of Linear Quadratic Regulator (LQR) Control System in Waypoint Flight Mission of Flying Wing UAV**, *Journal of Telecommunication, Electronic and Computer Engineering*, 12(4), 43–49, 2020. https://jtec.utem.edu.my/jtec/article/view/5696

[28] Gildardo Godinez-Garrido, et al., **Discrete Integral Optimal Controller for Quadrotor Attitude Stabilization: Experimental Results**, *MDPI (applied science)*, 13(16), 9293, 2023. https://doi.org/10.3390/app13169293

[29] Aline Ingabire, Andrey A. Sklyarov. **Control of longitudinal flight dynamics of a fixed-wing UAV using LQR**, **LQG and nonlinear control**, *E3S Web of Conferences*, 104, 02001, 2019. https://doi.org/10.1051/e3sconf/201910402001

[30] Trong Tu. **Synergistic Integration of LQR Control and PSO Optimization for Advanced Active Suspension Systems Utilizing Electro-Hydraulic Actuators and Electro-Servo Valves**, *EMITTER International Journal of Engineering Technology*, 13(1), 56-72, 2025. https://doi.org/10.24003/emitter.v13i1.892